

# On formulas for the velocity of Rayleigh waves in prestrained incompressible elastic solids

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**Abstract:** In the present paper, formulas for the velocity of Rayleigh waves in incompressible isotropic solids subject to a general pure homogeneous prestrain are derived using the theory of cubic equation. They have simple algebraic form and hold for a general strainenergy function. The formulas are concretized for some specific forms of strain-energy function. They then become totally explicit in terms of parameters characterizing the material and the prestrains. These formulas recover the (exact) value of the dimensionless speed of Rayleigh wave in incompressible isotropic elastic materials (without prestrain). Interestingly that, for the case of hydrostatic stress, the formula for the Rayleigh wave velocity does not depend on the type of strain-energy function. © 2010 by ASME.

**Author Keywords:** Incompressible; Prestrains; Prestresses; Rayleigh wave velocity; Rayleigh waves

**Index Keywords:** Cubic equations; Elastic materials; Elastic solids; Hydrostatic stress; Isotropic solids; Parameters characterizing; Pre-strain; Prestrains; Prestresses; Rayleigh wave velocity; Strain energy functions; Acoustic wave velocity; Strain energy; Wave propagation; Rayleigh waves

Year: 2010

Source title: Journal of Applied Mechanics, Transactions ASME

Volume: 77

Issue: 2

Page : 1-9

Link: Scopus Link

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ISSN: 218936

CODEN: JAMCA

DOI: 10.1115/1.3197139

Language of Original Document: English

Abbreviated Source Title: Journal of Applied Mechanics, Transactions ASME

Document Type: Article

Source: Scopus

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