Multiple solutions for a class of quasilinear elliptic equations of p(x)-Laplacian type with nonlinear boundary conditions

Chung N.T., Ng Q.-A.

Department of Mathematics and Informatics, Quang Binh University, 312 Ly Thuong Kiet, Dong Hoi, Quang Binh, Viet Nam; Department of Mathematics, College of Science, Vietnam National University, Hanoi, Viet Nam; Department of Mathematics, National University of Singapore, 2 Science Drive 2, 117543 Singapore, Singapore

Abstract: Using variational methods we study the non-existence and multiplicity of non-negative solutions for a class of quasilinear elliptic equations of p(x)-Laplacian type with nonlinear boundary conditions of the form -Div($| u|p^{(x)-2}u)+|u|^{p(x)-2}u=0$ in Ω | $u|^{p(x)-2}\backslash\partial u\backslash\partial$ n $=\lambda$ g(x,u) on $\partial \Omega$ where Ω is a bounded domain with smooth boundary, n is the outer unit normal to $\partial \Omega$ and λ is a parameter. Furthermore, we want to emphasize that g: $\partial \Omega \times [0,\infty) \to$ is a continuous function that may or may not satisfy the Ambrosetti-Rabinowitz-type condition. © 2010 Royal Society of Edinburgh.

Authors with affiliations:

• Chung, N.T., Department of Mathematics and Informatics, Quang Binh University, 312 Ly Thuong Kiet, Dong Hoi, Quang Binh, Viet Nam

• Ng, Q.-A., Department of Mathematics, College of Science, Vietnam National University, Hanoi, Viet Nam, Department of Mathematics, National University of Singapore, 2 Science Drive 2, 117543 Singapore, Singapore

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