Optimal adaptive sampling recovery

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Abstract: We propose an approach to study optimal methods of adaptive sampling recovery of functions by sets of a finite capacity which is measured by their cardinality or pseudo-dimension. Let W L_q, $0 < q \le \infty$, be a class of functions on I^d. For B a subset in L_q, we define a sampling recovery method with the free choice of sample points and recovering functions from B as follows. For each f W we choose n sample points. This choice defines n sampled values. Based on these sampled values, we choose a function from B for recovering f. The choice of n sample points and a recovering function from B for each f W defines a sampling recovery method S^B_n by functions in B. An efficient sampling recovery method should be adaptive to f. Given a family B of subsets in L_q, we consider optimal methods of adaptive sampling recovery of functions in W by B from B in terms of the quantity Denote $R_n(W,B)_q$ by $e_n(W)_q$ if B is the family of all subsets B of L_q such that the cardinality of B does not exceed 2^n, and by $r_n(W)_q$ if B is the family of all subsets B in L_q of pseudo-dimension at most n. Let $0 < p,q, \theta \le \infty$ and α satisfy one of the following conditions: (i) $\alpha > d/p$; (ii) $\alpha = d/p$, $\theta \le \min (1,q)$, $p,q < \infty$. Then for the d-variable Besov class $U^\alpha_{p,\theta}$ (defined as the unit ball of the Besov space $B^\alpha_{p,\theta}$), there is the following asymptotic order To construct asymptotically optimal adaptive sampling recovery methods for $e_n(U^\alpha_{p,\theta})_q$ and $r_n(U^\alpha_{p,\theta})_q$ we use a quasi-interpolant wavelet representation of functions in Besov spaces associated with some equivalent discrete quasi-norm.

Author Keywords: Adaptive sampling recovery; B-spline; Besov space; Quasi-interpolant wavelet representation

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