Non-linear sampling recovery based on quasi-interpolant wavelet representations

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Abstract: We investigate a problem of approximate non-linear sampling recovery of functions on the interval I:=[0,1] expressing the adaptive choice of n sampled values of a function to be recovered, and of n terms from a given family of functions Φ. More precisely, for each function f on I, we choose a sequence $\xi = \{\xi^s\}_{s=1}^{n}$ of n points in I, a sequence $a = \{a_s\}_{s=1}\}^{n}$ of n functions defined on ^{n} and a sequence $\Phi_n = \{V_{ks}\}_{s=1}^{n}$ of n functions from a given family Φ. By this choice we define a (non-linear) sampling recovery method so that f is approximately recovered from the n sampled values $f(\xi^1)$, $f(\xi^2)$,..., $f(\xi^n)$, by the n-term linear combination $S(f) = S(\xi, \Phi_n, a, f) := \sum_{s=1}^{n} a_s(f(\xi^1),...,f(\xi^n))V\{k_s\}$. In searching an optimal sampling method, we study the quantity $\nu_n(f, \Phi)_q := \{\Phi_n, \xi, a\}, ||F - S(\xi, \Phi_n, a, f)||_q$, where the infimum is taken over all sequences $\xi = \{\xi^s\}_{s=1}^{n}$ of n points, $a = \{a_s\}_{s=1}^{n}$ of n functions defined on ^{n}, and $\Phi n = \{V_{ks}\}\}_{s=1}^{n}$ of n functions from Φ. Let $U^{\alpha}_{p,\theta}$ be the unit ball in the Besov space $B^{\alpha}_{p,\theta}\}$ and M the set of centered B-spline wavelets $M_{k,s}(x) := N_r(2^k x + \rho - s)$, which do not vanish identically on I , where N_r is the B-spline of even order $r \geq [\alpha] + 1$ with knots at the points 0,1,...,r. For $1 \leq p,q \leq \infty$, $0 < \theta \leq \infty$ and $\alpha > 1$, we proved the following asymptotic order $\nu_n(U^{\alpha}_{p,\theta}, (f M)_q := \sup_{f \infty U \alpha p,\theta} \mu_n(f,M)_q\ n^{-\alpha}$. An asymptotically optimal non-linear sampling recovery method S^* for $\mu_n(U^{\alpha}_{p,\theta}, (f M)_q$ is constructed by using a quasi-interpolant wavelet representation of functions in the Besov space in terms of the B-splines $M_{k,s}$ and the associated equivalent discrete quasi-norm of the Besov space. For $1 \leq p < q \leq \infty$ the asymptotic order of this asymptotically optimal sampling non-linear recovery method is better than the asymptotic order of any linear sampling recovery method or, more generally, of any non-linear sampling recovery method of the form $R(H,\xi,f) := H(f(\xi^1),...,f(\xi^n))$ with a fixed mapping $H:^n$ to C(I) and n fixed points $\xi = \{\xi^s\}_{s=1}^{n}$.

Author Keywords: Adaptive choice; B-spline; Besov space; Non-linear sampling recovery; Quasi-interpolant wavelet representation

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