

# Analysis of trigonometric implicit Runge-Kutta methods

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**Abstract:** Using generalized collocation techniques based on fitting functions that are trigonometric (rather than algebraic as in classical integrators), we develop a new class of multistage, one-step, variable stepsize, and variable coefficients implicit Runge-Kutta methods to solve oscillatory ODE problems. The coefficients of the methods are functions of the frequency and the stepsize. We refer to this class as trigonometric implicit Runge-Kutta (TIRK) methods. They integrate an equation exactly if its solution is a trigonometric polynomial with a known frequency. We characterize the order and A-stability of the methods and establish results similar to that of classical algebraic collocation RK methods. © 2006 Elsevier B.V. All rights reserved.

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