Convolutions for the Fourier transforms with geometric variables and applications

Giang B.T., Mau N.V., Tuan N.M.

Dept. of Basic Science, Institute of Cryptography Science, 141 Chien Thang str., Thanh Tri dist., Hanoi, Viet Nam; Dept. of Mathematical Analysis, University of Hanoi, 334 Nguyen Trai str., Thanh Xuan dist. Hanoi, Viet Nam; Dept. of Math., Univ. of Edu, Ha Noi National Univ., G7 Build., 144 Xuan Thuy Rd., Cau Giay dist., Ha Noi, Viet Nam

Abstract: This paper gives a general formulation of convolutions for arbitrary linear operators from a linear space to a commutative algebra, constructs three convolutions for the Fourier transforms with geometric variables and four generalized convolutions for the Fourier-cosine, Fourier-sine transforms. With respect to applications, by using the constructed convolutions normed rings on $L_1(R^n)$ are constructed, and explicit solutions of integral equations of convolution type are obtained. Copyright © 2010 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim.
Author Keywords: Convolution; Factorization identity; Gaussian function; Generalized convolution; Hermite function

Authors with affiliations:

• Giang, B.T., Dept. of Basic Science, Institute of Cryptography Science, 141 Chien Thang str., Thanh Tri dist., Hanoi, Viet Nam

• Mau, N.V., Dept. of Mathematical Analysis, University of Hanoi, 334 Nguyen Trai str., Thanh Xuan dist. Hanoi, Viet Nam

• Tuan, N.M., Dept. of Math., Univ. of Edu, Ha Noi National Univ., G7 Build., 144 Xuan Thuy Rd., Cau Giay dist., Ha Noi, Viet Nam

References:

• Al-Musallam, F., Tuan, V.K., A class of convolution transforms (2000) Frac. Calc. Appl. Anal., 3 (3), pp. 303-314

• Arfken, G., Mathematical Methods for Physicists, 3rd edition (Academic Press, Orlando, 1985)Bateman, H., Erdely, A., Tables

of Integral Transforms (MC Gray-Hill, New York-Toronto-London, 1954)Bochner, S., Chandrasekharan, K., Fourier Transforms (Princeton University Press, Princeton, 1949)Britvina, L.E., Generalized convolutions for the Hankel transform and related integral operators (2007) Math. Nachr., 280 (9-10), pp. 962-970

- Britvina, L.E., A class of integral transforms related to the Fourier cosine convolution (2005) Integral Transforms Spec. Funct., 16 (5-6), pp. 379-389

- Brown, J.W., Churchill, R.V., and, Fourier Series and Boundary Value Problems, seven-th edition (McGraw-Hill Science/Engineering/Math, 2006)Cho, P.S., Kuterdem, H.G., Marks II, R.J., A spherical dose model for radio surgery plan optimization (1998) Biol. Med. Phys. Biomed. Eng., 43, pp. 3145-3148

- Churchill, R.V., Fourier Series and Boundary Value Problems (McGraw-Hill, New York, 1941)Garcia-Vicente, F., Delgado, J.M., Peraza, C., Experimental determination of the convolution kernel for the study of the spatial response of a detector (1998) Med. Phys., 25, pp. 202-207

- Garcia-Vicente, F., Delgado, J.M., Rodriguez, C., Exact analytical solution of the convolution integral equation for a general profile fitting function and Gaussian detector kernel (2000) Biol. Med. Phys. Biomed. Eng., 45 (3), pp. 645-650

- Giang, B.T., Mau, N.V., Tuan, N.M., Convolutions of the Fourier-cosine and Fourier-sine integral transforms and integral equations of the convolution type (2008) Herald of Polotsk State Uni., 50 (9), pp. 7-16

- Giang, B.T., Tuan, N.M., Generalized convolutions for the Fourier integral transforms and applications (2008) Journal of Siberian Federal Univ. Math. & Phys., 1 (4), pp. 371-379

- Giang, B.T., Tuan, N.M., Generalized onvolutions for the integral transforms of Fourier type and applications (2009) Frac. Calc. Appl. Anal., 12 (3), pp. 253-268

- Gohberg, I.S., Feldman, I.A., and, Convolution Equations and Projection Methods for their Solutions (Nauka, Moscow, 1971) (in Russian)Hochstadt, H., Integral Equations (John Wiley & Sons, Inc., 1973)Hömander, L., L2- estimates for Fourier integral operators with complex phase (1983) Ark. Mat., 21 (1), pp. 283-307

- Hömander, L., The Analysis of Linear Partial Differential Operators I (Springer-Verlag, Berlin - Heidelberg - Berlin - Tokyo, 1983)Kakichev, V.A., On the convolution for integral transforms (1967) Izv. ANBSSR, Ser. Fiz. Mat., 2, pp. 48-57. , in Russian

- Kakichev, V.A., On the matrix convolutions for power series (1990) Izv. Vuzov. Mat., 2, pp. 53-62. , in Russian

- Kakichev, V.A., Polyconvolution, Taganskij Radio-tekhnicheskij Universitet (1997). ISBN: 5-230-24745-2 (in Russian)Kakichev, V.A., Thao, N.X., Tuan, V.K., On the generalized convolutions for Fourier cosine and sine transforms (1998) East-West J. Math., 1 (1), pp. 85-90

- Naimark, M.A., Normed Rings (P. Noordhoff N. V., Groningen, Netherlands, 1959)Rudin, W., Functional Analysis (McGraw-Hill, New York-Hamburg-London-Paris-Tokyo, 1991)Thao, N.X., Khoa, N.M., On the generalized convolution with a weight function for the Fourier sine and cosine transforms (2006) Integral Transforms Spec. Funct., 17 (9), pp. 673-685

- Thao, N.X., Hai, N.T., and, Convolution for Integral Transforms and their Applications (Computer Center of the RAS, Moscow, 1997)Thao, N.X., Tuan, V.K., Hong, N.T., Generalized convolution transforms and Toeplitz plus Hankel integral equation (2008) Frac. Calc. App. Anal., 11 (2), pp. 153-174

- Thao, N.X., Tuan, V.K., Hong, N.T., Integral transforms of Fourier cosine and sine generalized convolution type (2007) Int. J. Math. Sci., 17, p. 11. , Art. ID 97250

- Thierry, C., Stolzenberg, G., The Wiener lemma and certain of its generalizations (1991) Bull. Amer. Math. Soc. (N. S.), New Ser., 24 (1), pp. 1-9

- Titchmarsh, E.C., Introduction to the Theory of Fourier Integrals (Chelsea Publishing Company, New York, N. Y., 1986)Tuan, N.M., Tuan, P.D., Generalized convolutions relative to the Hartley transforms with applications (2009) Sci. Math. Jpn., 70 (1), pp. 77-89

- Tuan, V.K., Integral transform of Fourier type in a new class of functions (1985) Dokl. Akad. Nauk BSSR, 29 (7), pp. 584-587. , in Russian
- Vilenkin, N.Y., Matrix elements of the indecomsable unitary representations for motion group of the Lobachevskii's space and generalized Mehler-Fox transforms Dokl. Akad. Nauk. UzSSR, 118 (2), pp. 219-222. , in Russian
- Vladimirov, V.S., Generalized Functions in Mathematical Physics (Mir Pub., Moscow, 1979)