The cohomology of the Steenrod algebra and representations of the general linear groups

Hung N.H.V.

Department Of Mathematics, VIetnam National University, Hanoi 334 Nguyen Trai Street, Hanoi, Viet Nam

Abstract: Let Tr_k be the algebraic transfer that maps from the coinvariants of certain GL_k-representations to the cohomology of the Steenrod algebra. This transfer was defined by W. Singer as an algebraic version of the geometrical transfer tr_k, : πS^* $((BV_k)+) \to \pi$ (S_0). It has been shown that the algebraic transfer is highly nontrivial, more precisely, that Tr_k, is an isomorphism for k = 1,2,3 and that Tr = $_k$ Tr_k is a homomorphism of algebras. In this paper, we first recognize the phenomenon that if we start from any degree d and apply Sq_0 repeatedly at most (k - 2) times, then we get into the region in which all the iterated squaring operations are isomorphisms on the coinvariants of the GL_k-representations. As a consequence, every finite S_q^0-family in the coinvariants has at most (k - 2) nonzero elements. Two applications are exploited. The first main theorem is that Tr_k is not an isomorphism for k ≥ 5. Furthermore, for every k > 5, there are infinitely many degrees in which Tr_k. is not an isomorphism. We also show that if Tr_ℓ detects a nonzero element in certain degrees of Ker(S_q^0). then it is not a monomorphism and further, for each k>l, Tr_k is not a monomorphism in infinitely many degrees. The second main theorem is that the elements of any S_q^0-family in the cohomology of the Steenrod algebra, except at most its first (k - 2) elements, are either all detected or all not detected by Tr_k, for every k. Applications of this study to the cases k = 4 and 5 show that Tr_4 does not detect the three families g, D_3 and p′, and that Tr_5 does not detect the family $\{h_{n+1}g_n | n \geq 1\}$.

Author Keywords: Adams spectral sequences; Invariant theory; Modular representations; Steenrod algebra

Year: 2005

Source title: Transactions of the American Mathematical Society

Volume: 357

Issue: 10

Page : 4065-4089

Correspondence Address: Hung, N.H.V.; Department Of Mathematics, VIetnam National University, Hanoi 334 Nguyen Trai Street, Hanoi, Viet Nam; email: nhvhung@vnu.edu.vn

ISSN: 29947

DOI: 10.1090/S0002-9947-05-03889-4

Language of Original Document: English

Abbreviated Source Title: Transactions of the American Mathematical Society

Document Type: Article

Source: Scopus

Authors with affiliations:

- Hung, N.H.V., Department Of Mathematics, VIetnam National University, Hanoi 334 Nguyen Trai Street, Hanoi, Viet Nam

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