Almost automorphic solutions of evolution equations

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Abstract: This paper is concerned with the existence of almost automorphic mild solutions to equations of the form (*) u̇(t)=Au(t) + f(t), where A generates a holomorphic semigroup and f is an almost automorphic function. Since almost automorphic functions may not be uniformly continuous, we introduce the notion of the uniform spectrum of a function. By modifying the method of sums of commuting operators used in previous works for the case of bounded uniformly continuous solutions, we obtain sufficient conditions for the existence of almost automorphic mild solutions to (*) in terms of the imaginary spectrum of A and the uniform spectrum of f.
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