

Massera's theorem for almost periodic solutions of functional differential equations

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Abstract: The Massera Theorem for almost periodic solutions of linear periodic ordinary differential equations of the form (*) $x' = A(t)x + f(t)$, where f is almost periodic, is stated and proved. Furthermore, it is extended to abstract functional differential equations (***) $x' = Ax + F(t)x_t + f(t)$, where A is the generator of a compact semigroup, F is periodic and f is almost periodic. The main techniques used in the proofs involve a new variation of constants formula in the phase space and a decomposition theorem for almost periodic solutions.

Author Keywords: Abstract functional differential equation; Almost periodic solutions; Decomposition; Massera's theorem; Variation of constants formula

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