On triviality of Dickson invariants in the homology of the Steenrod algebra

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Abstract: Let script A sign be the mod 2 Steenrod algebra and D_k the Dickson algebra of k variables. We study the Lannes-Zarati homomorphisms φ_k: $\text{Ext}_{\text{script A sign}}^{k,k+i}$(script F sign$_2$, script F sign$_2$ → (script F sign$_2$ script A sign$_0$ D_k)*$_i$, which correspond to an associated graded of the Hurewicz map H : π^{*S} (S^0) π^* ($Q_0 S^0$) → H*($Q_0 S^0$). An algebraic version of the long-standing conjecture on spherical classes predicts that φ_k = 0 in positive stems, for k > 2. That the conjecture is no longer valid for k = 1 and 2 is respectively an exposition of the existence of Hopf invariant one classes and Kervaire invariant one classes. This conjecture has been proved for k = 3 by Hu'ng. It has been shown that φ_k vanishes on decomposable elements for k > 2 and on the image of Singer's algebraic transfer for k > 2. in this paper, we establish the conjecture for k = 4. To this end, our main tools include (1) an explicit chain-level representation of φ_k and (2) a squaring operation Sq^0 on (script F sign$_2$ script A sign D_k)*, which commutes with the classical Sq^0 on $\text{Ext}_{\text{script A sign}}^{k}$(script F sign$_2$, script F sign$_2$) through the Lannes-Zarati homomorphism.

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